How to Solve System Dynamic’s Problems

A system dynamic’s problem involves two or more bodies (objects) under the influence of several “external” forces. The objects may ultimately rest, move with constant velocity, constant acceleration or some combination of these, but all objects of the system have the same motion. Another thing of importance is the distinction between internal and external forces.

1. Read the problem carefully! At this point you need to determine what objects comprise the system. An object usually belongs to the system if it is mass is known and / or it is known to be connected to another object. Generally, surfaces and the Earth itself are not considered a part of the system.

Read the following practice exercise and determine what objects make up the system.

i. Practice 1: A caterpillar tractor pushes a couple of car wrecks on a level junk yard. One car has a mass of 1500 kg. This car touches a truck with a mass of 2300 kg. The tractor applies 20 kN of force to the car. The coefficient of kinetic friction between both wrecks and the ground is 0.60. Determine the system acceleration and the force of the car on the truck and the truck on the car.

ANSWER: The car and the truck comprise this system. We exclude the tractor as we do not know its mass. We exclude the surface for essentially the same reason.

ii. Practice 2: Using the diagram provided determine (a) which way the system moves, (b) if the system spontaneously moves, (c) if it were in motion its acceleration and (d) if it were in motion the tension in all connecting strings.

ANSWER: M1 and M2 are the system, as we know their masses and they are connected by a common string. The surfaces of the incline and the Earth will be outside this system.
2. Next you need to determine the direction of travel for the system and the location of the “x-axis.” You may need to consider this object by object especially if each object is on a different surface. However, in general terms all objects will move either to the left or right.

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ANSWER: We are told the yard is level so the ‘x-axis’ is horizontal. In this problem it is entirely up to you to have the tractor push right or left but it must touch the car. I have arbitrarily chosen right.

ii. Practice 2: Using the diagram provided determine (a) which way the system moves, (b) if the system spontaneously moves, (c) if it were in motion its acceleration and (d) if it were in motion the tension in all connecting strings.

\[ \mu_s = 0.10 \text{ and } \mu_k = 0.02, \text{ for all surfaces} \]

ANSWER: Both masses can only move along the incline plane, therefore the ‘x-axis’ will be tilted for each mass. As for direction of travel we have two options either it will be obvious or not. It usually not obvious if one or more objects are on an inclined plane.

So what you do when it is not obvious is calculate the external forces on the objects that could make them move. Ignore all frictions as they will not cause motion. Ignore forces or components perpendicular to the potential directions of
travel. Ignore connecting forces as they are internal. In this example, it is only the x-components of gravity that can cause motion. You now must determine the size of all of these and add up the ones pointing right, and then add up the ones pointing left. Which ever side is bigger determines the direction of travel.

Since \( F_{g2x} > F_{g1x} \), then M2 moves down hill while M1 moves up hill.

3. In practice 2 we have an additional challenge of deciding if an initially resting system will spontaneously move. This requires solving for the system acceleration. However, if a friction will be used in the net force equation it will only be static friction. You should now calculate the other external forces such as normal force and static friction. Once you know the values of these external forces solve for the system acceleration with the net force equation. Again only use forces along the x-axis to calculate acceleration.

The value of the acceleration determines the answer. If ‘a’ is negative or zero, then the system stays at rest. If ‘a’ is positive, it will spontaneously move.

i. Practice 2: Using the diagram provided determine (a) which way the system moves, (b) if the system spontaneously moves, (c) if it were in motion its acceleration and (d) if it were in motion the tension in all connecting strings.

\[ \mu_s = 0.10 \text{ and } \mu_k = 0.02, \text{ for all surfaces} \]

\[ F_{net} = \sum F_{with} - \sum F_{against} \]

\[ a = \frac{\sum F_{with} - \sum F_{against}}{m_t} \]

\[ a = \frac{F_{g2x} - F_{g1x} - F_{s1} - F_{s2}}{m_t} \]

\[ a = \frac{(72.3 - 50.2 - 13.8 - 5.06)N}{24.0kg} = 0.135 \text{ m/s}^2 \]

This system spontaneously accelerates.
4. In both practices, we must find the acceleration assuming that somehow they got in motion. If solving for the system acceleration and frictions are involved only be kinetic friction will appear in the net force equation. NEVER USE STATIC AND KINETIC FRICTION AT THE SAME TIME. Only use forces on the x-axis to solve for the acceleration.

The value of the acceleration determines the nature of the motion. If ‘a’ is negative then the system slows down and stops. It most likely will remain stopped unless the static friction is really low. If ‘a’ is zero, then the system moves with constant velocity in the desired direction of travel. If ‘a’ is positive, the system will accelerate.

i. Practice 1: A caterpillar tractor pushes a couple of car wrecks on a level junk yard. One car has a mass of 1500 kg. This car touches a truck with a mass of 2300 kg. The tractor applies 20 kN of force to the car. The coefficient of kinetic friction between both wrecks and the ground is 0.60. Determine the system acceleration and the force of the car on the truck and the truck on the car.

\[ F_1 = 20000 \text{ N} \]

Fricion

\[ F_{k1} = 0.6(1500)(9.81) \]
\[ F_{k1} = 8829 \text{ N} \]

\[ F_{k2} = 0.6(2300)(9.81) \]
\[ F_{k2} = 13538 \text{ N} \]

\[ F_{\text{net}} = m_i a = \sum F_{\text{with}} - \sum F_{\text{against}} \]
\[ a = \frac{\sum F_{\text{with}} - \sum F_{\text{against}}}{m_i} \]
\[ a = \frac{F_1 - F_{k1} - F_{k2}}{m_i} \]
\[ a = \frac{(20000 - 13538 - 8829)N}{3800kg} = -0.635 \text{ m/s}^2 \]

This system is slowing down to a halt.
ii. Practice 2: Using the diagram provided determine (a) which way the system moves, (b) if the system spontaneously moves, (c) if it were in motion its acceleration and (d) if it were in motion the tension in all connecting strings.

\[ \mu_s = 0.10 \text{ and } \mu_k = 0.02, \text{ for all surfaces} \]

\[ F_{k1} = 0.02(15)(9.81)\cos20 \]
\[ F_{k1} = 2.8 \text{ N} \]

\[ F_{g1x} = 50.2 \text{ N} \]

\[ M1 = 15 \text{ kg} \]
\[ F_{s2} = 0.02(9)(9.81)\cos55 \]
\[ F_{s2} = 1.0 \text{ N} \]

\[ M2 = 9 \text{ kg} \]
\[ F_{g2x} = 72.3 \text{ N} \]

\[ F_{\text{net}} = m, a = \Sigma F_{\text{with}} - \Sigma F_{\text{against}} \]

\[ a = \frac{\Sigma F_{\text{with}} - \Sigma F_{\text{against}}}{m} \]

\[ a = \frac{F_{g2x} - F_{g1x} - F_{k1} - F_{k2}}{m} \]

\[ a = \frac{(72.3 - 50.2 - 2.8 -1.0)N}{24.0kg} = 0.758/m^2 \]

This system is speeding up.
5. Finding connecting forces in any problem requires selecting one object and solving the net force equation for it. Use the system acceleration calculated in the last step.

i. Practice 1: A caterpillar tractor pushes a couple of car wrecks on a level junk yard. One car has a mass of 1500 kg. This car touches a truck with a mass of 2300 kg. The tractor applies 20 kN of force to the car. The coefficient of kinetic friction between both wrecks and the ground is 0.60. **Determine** the system acceleration and **the force of the car on the truck and the truck on the car**.

![Diagram of a caterpillar tractor pushing a couple of car wrecks on a level junk yard.]

Using the truck as the target object:

\[ F_{net} = m_2a = \Sigma F_{with} - \Sigma F_{against} \]
\[ 2300(-0.635 \text{ m/s}^2) = F_{\text{car on truck}} - F_{k2} \]
\[ F_{\text{car on truck}} = (-1461 + 13538)N = 12077N \]

By Newton's 3rd Law, the force of the truck on the car has the same value in the opposite direction.
ii. Practice 2: Using the diagram provided determine (a) which way the system moves, (b) if the system spontaneously moves, (c) if it were in motion its acceleration and (d) if it were in motion the tension in all connecting strings.

\[ \mu_s = 0.10 \text{ and } \mu_k = 0.02, \text{ for all surfaces} \]

\[ F_{k1} = 0.02(15)(9.81)\cos20 \]
\[ F_{k1} = 2.8 \text{ N} \]

\[ F_{g1x} = 50.2 \text{ N} \]

Using M1 as the target object:

\[ F_{\text{net}} = m_1 a = \Sigma F_{\text{with}} - \Sigma F_{\text{against}} \]

\[ 15(0.758) = F_T - F_{g1x} - F_{k1} \]

\[ F_T = 11.37 + F_{g1x} + F_{k1} \]

\[ F_T = (11.37 + 50.2 + 2.8)N = 64.4N \]